Nonlocal Boxes Seen as Tensors and Used to Determine if a Physical Theory is Realistic or Not

References: PRL 132,070201 (2024) [1], Quantum 8,1402 (2024) [2], arXiv:2406.02199 [3].

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Motivation

Observations

- Quantum Mechanics Theory ($\mathcal{Q}$): validated by many experiments.
- Non-Signalling Theory ($\mathcal{NS}$): violates a fundamental mathematical principle $\Rightarrow$ "unrealistic".

Question

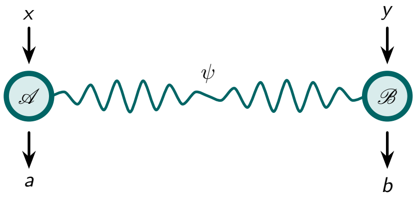
What is the largest theory T such that $\mathcal{Q} \subseteq T \subseteq \mathcal{NS}$ and that is realistic?

1. Correlation Set Induced by a Theory

Definition. The *correlation set* is the set $\mathcal{P} \subseteq \mathbb{R}^A \otimes \mathbb{R}^B \otimes \mathbb{R}^X \otimes \mathbb{R}^Y$ of all tensors $\mathbf{P}$ whose entries are conditional probability distributions as follows:

- The coordinates of $\mathbf{P}$ are denoted by "$\mathbf{P}(a, b \,|\, x, y)$" with $a \in \{1, .., A\}$, $b \in \{1, .., B\}$, $x \in \{1, .., X\}$, $y \in \{1, .., Y\}$;
- $\forall a, b, x, y, \quad \mathbf{P}(a, b \,|\, x, y) \geqslant 0$;
- $\forall x, y, \quad \sum_{ab} \mathbf{P}(a, b \,|\, x, y) = 1$.

Definition. Given a physical theory T, its *induced correlation set* is the subset $\mathcal{P}_T \subseteq \mathcal{P}$ of all probabilities $\mathbf{P}(a, b \,|\, x, y)$ obtainable from measuring a bipartite state $\psi \in T$.

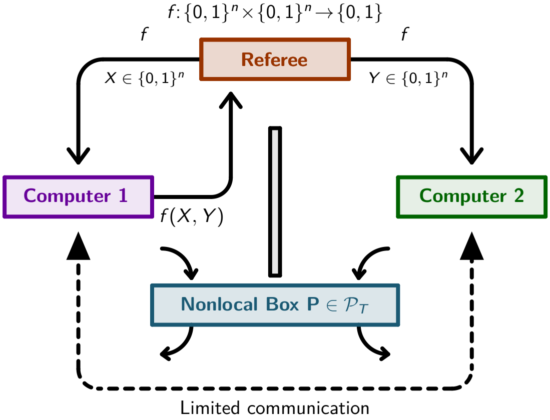


Examples.
- $\mathcal{P}_{\text{quantum}} := \mathcal{Q} := \Big\{ \mathbf{P} \in \mathcal{P} \,:\, \exists \text{ POVM } A_{a|x}, B_{b|y} \text{ such that } \mathbf{P}(a, b \,|\, x, y) = \langle \psi \,|\, A_{a|x} \otimes B_{b|y} \,|\, \psi \rangle \Big\}$.
- $\mathcal{P}_{\text{non-sign.}} := \mathcal{NS} := \Big\{ \mathbf{P} \in \mathcal{P} \,:\, \sum_a \mathbf{P}(a, b|x, y) = \sum_a \mathbf{P}(a, b|x', y) \text{ and } \sum_b \mathbf{P}(a, b|x, y) = \sum_b \mathbf{P}(a, b|x, y') \Big\}$.

Definition. A tensor $\mathbf{P} \in \mathcal{P}_T$ is called *non-local box*.

2. Prove that a Theory is Unrealistic

Communication Complexity:



Principle of Communication Complexity (CC). It is not possible that only one bit of communication is enough to compute any function f with arbitrary large input size n.

Axiom. If a theory T violates CC, then it is said *unrealistic*.

Examples. ■ $\mathcal{Q}$ does not violate CC,
■ $\mathcal{NS}$ violates CC $\Rightarrow$ unrealistic.

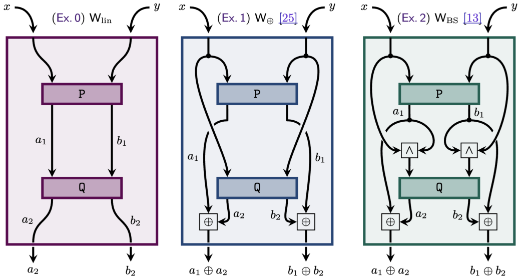
Question. What is the largest theory T such that $\mathcal{Q} \subseteq \mathcal{P}_T \subseteq \mathcal{NS}$ and that is realistic according to CC?

3. Examples of Techniques

Case $\mathbf{A} = \mathbf{B} = \mathbf{X} = \mathbf{Y} = 2$: "CHSH game scenario." Fix a theory T such that $\mathcal{Q} \subseteq \mathcal{P}_T \subseteq \mathcal{NS}$ containing a nonlocal box $\mathbf{P} \in \mathcal{P}_T$ with good properties (high CHSH value).

Technique 1 (PRL 132,070201 (2024)**).** Find a protocol using this nonlocal box $\mathbf{P}$ and majority vote to amplify the success of the computers in guessing the value $f(X, Y)$ for any f. Deduce that there is a violation of CC, and that T is unrealistic.

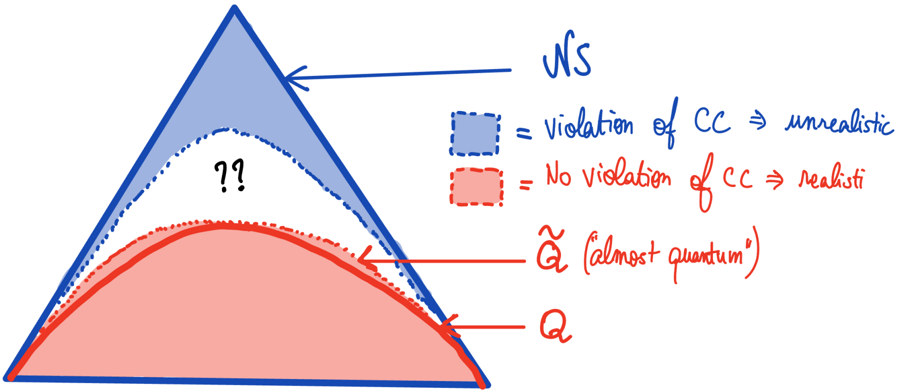
Technique 2 (Quantum 8,1402 (2024)**).** Wire the nonlocal box $\mathbf{P}$ with itself to generate a new nonlocal box that collapses CC. Therefore T implies the violation of CC and is unrealistic.



Case $\mathbf{A} = \mathbf{B} = \mathbf{X} = \mathbf{Y} \geqslant 2$:

Technique 3 (arXiv:2406.02199**).** Use a nonlocal game based on graph isomorphism and go back to the CHSH game (Techniques 1 and 2). Deduce the violation of CC.

4. What is Currently Known



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