Quantum XOR games via tensor norms

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Random Tensors and Related Topics

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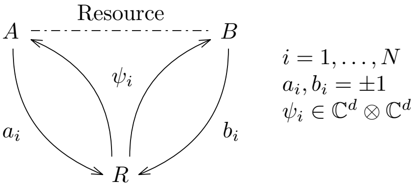

The game is defined by:

- A family of bipartite states: $|\psi_i\rangle \in \mathbb{C}^d \otimes \mathbb{C}^d$, $i = 1, \cdots, N$
- a prob. distribution $\pi : [N] \to [0,1]$ (questions)
- Some coefficients $c_i \in \{-1, 1\}$ for every $i = 1, \cdots, N$

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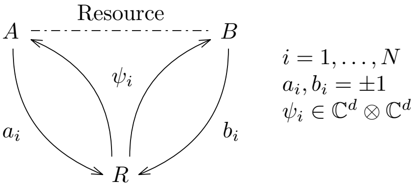

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It is clear that $P_{win}(G) \geq \frac{1}{2}$. Hence, one usually works with the bias:

$$\beta(G) := 2\big(P_{win}(G) - P_{random}(G)\big) \in [0, 1].$$

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A strategy to play a qXOR game is described by a linear map $\mathcal{P} : \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B) \to \mathbb{R}^4_+$ such that, for any given state ρ acting on $\mathcal{H}_A \otimes \mathcal{H}_B$, it assigns a probability distribution over the questions:

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$$\begin{aligned}\mathcal{P}(\rho) &= \mathcal{P}(a, b|\rho)_{a,b=\pm 1} \\ &= tr(\mathcal{M}_{a,b}\rho)_{a,b=\pm 1},\end{aligned}$$

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$$\beta(G; \mathcal{P}) = 2\big(P_{win}(G) - P_{random}(G)\big) = \sum_{i=1}^{N} p_i c_i \sum_{a,b=\pm 1} abP(a, b|\rho_i)$$

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$$= tr\left(\overbrace{\left(\sum_{a,b=\pm 1} ab\mathcal{M}_{a,b}\right)}^{\gamma \in S_\infty(\mathcal{H}_A)\otimes S_\infty(\mathcal{H}_B)} \overbrace{\left(\sum_{i=1}^{N} p_i c_i \rho_i\right)}^{G \in S_1(\mathcal{H}_A)\otimes S_1(\mathcal{H}_B)} \right) = tr(\gamma \cdot G)$$

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$$\mathcal{M}_{a,b} = tr_{A'B'}\left((E_a^{AA'} \otimes F_b^{BB'})(1_{AB} \otimes \rho_{A'B'})\right) \in \mathcal{B}(\mathcal{H}_A) \otimes \mathcal{B}(\mathcal{H}_B)$$

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- For every qXOR game $\beta^*_{max}(G) \leq 2\sqrt{2}\beta(G)$

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Remark: It is not enough to control $\beta^*_{max}(G)$

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Everything can be stated analogously to the bipartite case. But we need to work with tensors now:

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Useful (sometimes $\cdots$): LOCC strategies $\subset$ SEP strategies

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Theorem (Junge, P.): Given natural numbers d and m s.t. $d \geq Cm^4\sqrt{\log m}$ (C is a universal constant) we have, for large enough m,

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Corollary: There exists a family of tripartite XOR games $\{G_n\}_n$ s.t.

$$\lim_n \frac{\beta^*(G_n)}{\beta_{LOCC}(G_n)} = \infty.$$

THANK YOU VERY MUCH